**Math 3305 Chapter 3 Section 3 Script**

**The Pythagorean Theorem**

There are MANY proofs of the Pythagorean Theorem. In 1947 [The Pythagorean Proposition](http://www.cut-the-knot.org/pythagoras/index.shtml#Loomis) was printed by Professor Elisha Scott Loomis. The book is a collection of 367 proofs of the Pythagorean Theorem and was republished by NCTM in 1968.

Theorem 3.3.1

If triangle ABC is a triangle with a right angle at C, then .

We’ll look at only one proof.



Now let’s redo these into similar triangles with the right angle in the lower left. Rename

AD = C1 and DB = C2. So C = C1 + C2. Label sides “a” and “b” across from these angles.

We will use AA Similarity three times!

Note that a/c = c2/a and c/b = b/c1. Let’s go from there.



**Converse of the Pythagorean Theorem**

PT: If triangle ABC is a triangle with a right angle at C, then  .

Page 7:  original;  converse.

Converse: If triangle ABC satisfies , then angle C is a right angle.

**Proof of converse:** given a triangle where the equation is true…show that C is a right triangle in the original triangle.

Construct a second triangle with the same leg lengths and C’ is right.

Now, use the equation with the second coordinates a’, b’, and c’ via the PT.

Substitute with the hypothesis equation on the right a and b for the primed…substitute c on the right…c = c’…SSS congruence. Now C is right by CPCF.

**3.3 Essay One**

Write out the inverse and the contrapositive to the Pythagorean Theorem. Discuss the truth value of each with illustrations.

Now let’s change topics to Trigonometry for just a little bit.

**Trigonometry** – page 110

Given a right triangle, let’s map out the 6 trig functions by relationship.

Trigonometric Ratios - know by heart!

Given a right triangle, the following trigonometric ratios are defined:



“SOHCAHTOA”



Each trig function has a “co” function. And note each “s” is associated with a “c” in the first two.

**Popper 3.3 Question 1**

The cofunction for cosine is cosecant.

A. True

B. False

Here’s a Pythagorean Triple triangle: Everybody WANTS it to be a 30-60-90 but let’s actually check it using the cosine inverse function on your calculator.



Let’s get the Big Three sine, cosine, and tangent. Let’s look at “Trig inverse” on our calculators and get the angle measures.



We’ll look at 30-60-90s in a bit!

Note the one in the book: 5 – 12 – 13, bottom page 110

Pythagorean Triples! 7-24-25 and 9-40-41 are a couple more.

Most right angles do not have natural numbers for all 3 side lengths.

TPT can be applied to situations when the unknown is something other than the hypotenuse. Solve for x:

x

cm

m



BAC

=

90.00



C

B

A

9cm

Let’s use our calculators to get the measure of angle B and angle A

And now a discussion of the other angles: β & 

Note:  COMPLEMENTS not supplements



**3.3 Essay Question Two**: Why is sin(B) equal to cos(theta)?

The angles and are ACUTE angles of the right triangle. A is the right angle.

There can only be one right angle or obtuse angle in a triangle in EG!

What is a formula for the measure of angle 

(n.b. It’s not independent of ∠!)

**3.3 Popper Question Two**

If we have a 90 degree angle in a Spherical Triangle, is it true that both of the other two angles are always acute, i.e. less than 90 degrees?

A. Yes B. No

**Two Special Triangles:**

Note that **30° - 60° - 90°** go together in a right triangle.

**Isosceles right triangles** are the second special triangles.

Mnemonic: **30° - 60° - 90°** :: small, medium, big :: *x*, , 2*x*

**(note: )**



Check those sides? Are they right? We have a theorem about this!

sin 30° sin 60°

inverse sine of the sine…

cos 30° cos 60°

inverse cosine of the cosine…

tan 30° tan 60°

If you have an isosceles triangle with side length 6 cm, how can a 30-60-90 triangle help you with the length of the altitude?

What is the height of the triangle?



and **45° - 45° - 90°** go together in an **isosceles right triangle**.

Check those side lengths! Are they right?

Note the **hypotenuse** is the longest side in any right triangle

**Note:** 





sin 45° cos 45°

inverse sine inverse cosine

tan 45°

**Popper 3.3 Question 4**

Given an isosceles right triangle with a hyp otenuse of 5cm, what is the leg length?

A.  B.  C.  D. 

**MORE VOCABULARY:**

The angles 30°, 45°, and 60° are all called “**reference angles**”.

0°, 90°, 180°, 270°, and 360° are all called “**quadrantal angles**”.

Angles ON the axes, not IN a particular quadrant

Ms. Leigh’s Famous Chart for remembering sine and cosine:

Fill in radian measure in row 2. Then starting with row 3:

Count off left to right starting with 0.

Count back right to left starting with 0 in row 4.

Square root and divide by 2 in rows 3 and 4.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| angle in deg | 0° | 30° | 45° | 60° | 90° |
| angle in rad |  |  |  |  |  |
| sine |  |  |  |  |  |
| cosine |  |  |  |  |  |
| tangent |  |  |  |  |  |

**3.3 Ms. Leigh’s Problem 1.1** fill in and turn in with homework!

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| --- | --- | --- | --- | --- | --- |
| angle in deg | 0° | 30° | 45° | 60° | 90° |
| angle in rad |  |  |  |  |  |
| sine |  |  |  |  |  |
| cosine |  |  |  |  |  |
| tangent |  |  |  |  |  |

And the Supplemental Angle Conundrum with the Sine Function.

Let’s look at some supplements

70 degrees and 110 degrees

Sin(70) Sin(110) Sin inverse

Sin(50) Sin(120) Sin inverse

So, you need to apply some other knowledge like if it’s acute or obtuse to finish a problem!

**3.3 Ms. Leigh’s homework 2**

Sketch a pair of vertical angles. NOT 90/90. Measure them and make sure they are supplement. Using your calculator find the sine of each angle measure. And using  on your calculator on the sine value. See what you get. Come up with a memory aid to help you remember that there’s an obtuse angle with the SAME sine value.

Wrapping up:

A 4 question popper and two essays PLUS

3.3 #6 AND 4 Ms. Leigh Questions: two in the text and two next page.

**3.3 Ms. Leigh’s Problem Three**

Suppose we have the following scenario. What is BC? Use the Law of Cosines!

**3.3 Ms. Leigh’s Problem Four** Use the Law of Cosines to FIND cos(A). Then use cosine inverse to get the measure of angle A.

What is the cos(A)?

Hints: fill in the formula with what you know and BACKSOLVE for cos A!

How will we find the measure of angle A? hint use your calculator!